Note Title 4/18/2019

Ex $X(s) = \dfrac{s+1}{s^2+5s+6} = \dfrac{s+1}{(s+2)(s+3)} = \dfrac{A}{s+2} + \dfrac{B}{s+3}$

$A = (s+2)\left(\dfrac{s+1}{(s+2)(s+3)}\right)\Big|_{s=-2} = \dfrac{s+1}{s+3}\Big|_{s=-2} = \dfrac{-2+1}{-2+3} = \dfrac{-1}{1} = -1$

$B = (s+3)\left(\dfrac{s+1}{(s+2)(s+3)}\right)\Big|_{s=-3} = \dfrac{s+1}{s+2}\Big|_{s=-3} = \dfrac{-3+1}{-3+2} = 2$

$\therefore X(s) = \dfrac{-1}{s+2} + \dfrac{2}{s+3}$

$e^{-bt}u(t) \Leftrightarrow \dfrac{1}{s+b}$

$\therefore x(t) = -e^{-2t}u(t) + 2e^{-3t}u(t) = \left(2e^{-3t} - e^{-2t}\right)u(t)$

Ex

$$X(s) = \frac{s^2 - 2s + 1}{s^3 + 3s^2 + 4s + 2}$$

$$A(s) = s^3 + 3s^2 + 4s + 2 = (s + 1 - j)(s + 1 + j)(s + 1)$$

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>> N=[1 -2 1]

N =

    1    -2     1

>> D=[1 3 4 2]

D =

    1     3     4     2
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>> [r,p]=residue(N,D)

r =

  -1.5000 + 2.0000i
  -1.5000 - 2.0000i
   4.0000 + 0.0000i


p =

  -1.0000 + 1.0000i
  -1.0000 - 1.0000i
  -1.0000 + 0.0000i
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$$A(s) = s^3 + 3s^2 + 4s + 2 = (s+1-j)(s+1+j)(s+1)$$

$$X(s) = \frac{A}{(s+1-j)} + \frac{B}{s+1+j} + \frac{C}{s+1}$$

$$A = (s+1-j)\left(\frac{s^2-2s+1}{(s+1-j)(s+1+j)(s+1)}\right)\Bigg|_{s=-1+j}$$

$$A = \frac{s^2-2s+1}{(s+1+j)(s+1)} = \frac{(-1+j)^2-2(-1+j)+1}{(-1+j+1+j)(-1+j+1)}$$

$$= \frac{1-2j-1+2-2j+1}{(2j)(j)} = \frac{3-4j}{-2} = -1.5+2j$$

$$B = A^* = -1.5-j2$$

$$C = (s+1)\left(\frac{s^2-2s+1}{(s+1-j)(s+1+j)(s+1)}\right)\Bigg|_{s=-1}$$

$$= \frac{(-1)^2 - 2(-1)+1}{(-1+1-j)(-1+1+j)} = \frac{1+2+1}{(-j)(j)} = 4$$

$$X(s) = \frac{-1.5+2j}{(s+1-j)} + \frac{-1.5-2j}{(s+1+j)} + \frac{4}{s+1}$$

$$x(t) = (-1.5+j2)e^{-(1-j)t} + (-1.5-j2)e^{-(1+j)t} + 4e^{-t}, \quad t \geq 0$$

OR

$$x(t) = 5e^{-t}\cos(t + 126.87^\circ) + 4e^{-t}, \quad t \geq 0$$

Ex

$$X(s) = \frac{5s - 1}{s^3 - 3s \div 2}$$

$$X(s) = \frac{5s-1}{(s-2)(s+1)^2} = \frac{A}{(s+1)} + \frac{B}{(s+1)^2} + \frac{C}{s-2}$$

$$C = (s-2)\left(\frac{5s-1}{(s-2)(s+1)^2}\right)\Big|_{s=2} = \frac{10-1}{(2+1)^2} = \frac{9}{9} = 1$$

$$B = (s+1)^2\left(\frac{5s-1}{(s-2)(s+1)^2}\right)\Big|_{s=-1} = \frac{-5-1}{-1-2} = \frac{-6}{-3} = 2$$

$$A = \frac{1}{1}\left\{\frac{d}{ds}(s+1)^2\left(\frac{5s-1}{(s-2)(s+1)^2}\right)\Big|_{s=-1}\right\}$$

$$= \frac{d}{ds}\left(\frac{5s-1}{s-2}\right)\Big|_{s=-1}$$

$$= \frac{(s-2)(5) - (5s-1)(1)}{(s-2)^2}\Big|_{s=-1} = \frac{5s - 10 - 5s + 1}{(s-2)}\Big|_{s=-1}$$

$$= \frac{-9}{(s-2)^2}\Big|_{s=-1} = \frac{-9}{(-1-2)^2} = \frac{-9}{9} = -1$$

$$X(s) = \frac{-1}{s+1} + \frac{2}{(s+1)^2} + \frac{1}{s-2}$$

$$x(t) = -e^{-t} + 2te^{-t} + e^{2t}, \quad t \geq 0$$